

ANALYSIS

Edited by
BERNARD MAYO
with the advice of

A. J. Ayer	P. T. Geach
R. B. Braithwaite	C. A. Mace
Herbert Dingle	A. M. MacIver
A. E. Duncan-Jones	H. H. Price

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THE SYMBOLISING OF NATURAL DEDUCTION

By J. L. MACKIE

I. The purpose of this paper.

The methods used in standard textbooks, such as Quine's *Methods of Logic* and Copi's *Symbolic Logic*, to symbolise arguments employing the processes of natural deduction¹ seem to be inelegant in certain ways and for that reason difficult to understand and teach.

It would appear to be a requirement for elegance in a method of deduction that the decision whether a certain formula follows validly or does not follow from another formula should depend wholly on the formal features of the two formulae themselves. Another requirement is that each formula should be unambiguous, that the interpretation of its structure should not depend upon its context. A third requirement is that a deduction should be valid in each line, and thus capable of being checked step by step, that we should not have to look back over the whole argument to decide whether it is valid as a whole. And a fourth requirement is that the system should be free from arbitrary rules or restrictions, that every rule should be easily recognisable as justified by the meaning of the symbols used.

These requirements are satisfied fairly well by, say, traditional logic or by deduction within the propositional calculus, but they are not satisfied by the current presentations of natural deduction. Thus in Copi's presentation it may or may not be a valid inference if we go from Fw to $(x)Fx$: it is valid only if w is an "arbitrarily selected individual", so that this universal generalisation is valid if, for example, w was introduced by universal instantiation, but not if w first occurred as an individual name or was introduced by existential instantiation. Similarly Copi lays it down that universal generalisation is not permitted with regard to a symbol that has had a free occurrence in any previous formula that was introduced by existential instantiation, or within the scope of an assumption with regard to a symbol that has had a free occurrence in that assumption.²

That is, we have to look back at the earlier history of each symbol in the argument before we can tell what we are allowed to do next. The same results are achieved by Quine by means of

¹ "Universal instantiation" (U.I.), "universal generalisation" (U.G.), "existential generalisation" (E.G.), and "existential instantiation" (E.I.).

² See Copi, *Symbolic Logic*, pp. 104-106.

a convention about the order in which new letters are to be introduced together with the device of "flagging" a symbol used in E.I. or U.G., and the rules that no symbol may be flagged twice and that no symbol flagged in the course of a deduction may be free in the last line or in any premise of the last line.¹ But these rules also violate the above-stated requirements of elegance: they appear arbitrary, and are justified as *ad hoc* devices to prevent certain fallacies.

I have tried to show in another article² that these rules and restrictions are not really arbitrary: they can be explained and understood. But their justification is not obvious from the meaning of the symbols themselves, and, as I have argued in that article, the restrictions are needed because various formulae are in themselves ambiguous.

It might be worth while, then, to revise the symbolism in such a way as to remove the ambiguities, to make the formulae as far as possible self-explanatory, and thus to satisfy the above-stated requirements of elegance.

II. Variables and constants.

As a preliminary to such a revision we need to clarify the distinction between a "variable" and a "constant" and the notion of a "free" variable.

If in ordinary conversation we say "Socrates was a Greek philosopher" we are talking about the actual historical individual, Socrates; the word "Socrates" is being used as the name of an individual. But if in teaching elementary logic we say "All men are mortal, Socrates is a man, therefore Socrates is mortal", it is not quite so clear what we are doing. Probably we are using "Socrates" just as a stock name, by means of which to convey the logical truth that if x is any individual then "All men are mortal, x is a man, therefore x is mortal" is a valid argument. We are not talking about Socrates, because we could convey exactly what we want to convey by using the name "Smith" instead. Are we to say, then, that "Socrates" is here used as a variable? This might be misleading, because it is a distinguishing feature of this particular argument form that the minor term is the name of an individual: "Socrates" does not function in the minor premise or in the conclusion here in the way in which x functions in (x) $[Fx \supset Gx]$. The difference is that x is a variable *within the example*: the meaning of this formula involves

¹ See Quine, *Methods of Logic*, pp. 160-166.

² "The Rules of Natural Deduction", *ANALYSIS* 19.2 (December 1958), pp. 27-35.

the notion of x running through the whole series of its possible values, or at any rate of our being able to replace x in $Fx \supset Gx$ by any one of these values. By contrast "Socrates" *within the example* is not a variable but a constant: within this argument "Socrates" is being treated as the name of some definite individual. But when we step outside the example and consider it as an example, when we treat it as a specimen argument, as indicating an argument form, then "Socrates" becomes a variable, something to be replaced in each concrete argument that is of this form by any one of a whole range of individual names.

When we use this example to indicate an argument form, what we want to convey is a universal statement: we are saying that all arguments of this form are valid. That is, there is an implicit quantification, but it applies to the argument as a whole. What lies within the scope of this implicit quantification is not the minor premise or the conclusion taken by itself, but the whole argument: we are saying "For all x , 'All men are mortal, x is a man, therefore x is mortal' is valid".

Similarly if we say that $p \supset q$, p , $\therefore q$ is valid, we may have laid it down that p and q are shorthand for some definite propositions like "The sky is blue" and "Air reflects light", and then we shall be asserting the validity of a certain concrete argument. Alternatively we may be using this as an argument form, in which case what we want to convey has an implicit double quantification, "For all p , for all q , if p and q are propositions, ' $p \supset q$, p , $\therefore q$ ' is valid". Here too the symbols p and q function as variables when we are looking at this as an argument form, but they can function as constants, as indications of definite propositions, within the example.

This situation may be compared with what we do in mathematics when we give a formula for, say, a differentiation, such as $d/dx(ax^2 + bx + c) = 2ax + b$. Here x is a variable within the example; a , b and c are constants within the example, though when we use the formula we treat them as variables, replacing them by the values needed for some particular application.

Another way of using a symbol is to use it to indicate an "arbitrarily selected individual". As I have shown in the above-mentioned article, the formula ϕy , where y is an arbitrarily selected individual, represents the set of formulae ϕa , ϕb , ϕc , etc., where a , b , c , etc., are all the actual individuals. A symbol so used is implicitly universally quantified within the example. It plays a part *within the example* similar to that which "Socrates"

plays when the example within which it occurs is looked at from the outside as a specimen argument, as indicating an argument form. Thus an "arbitrarily selected individual" symbol is implicitly what an x bound by a quantifier is explicitly, a variable within the example.

A "bound" variable is being explicitly used as a variable within the example; but what is a "free" variable? This phrase may refer to any one of the following three quite different ways in which a symbol can be used:

- (i) Suppose that we put the above-quoted syllogism into symbols as $(x) [Fx \supset Gx], Fa, \therefore Ga$. Then in the second premise, Fa , a is doing the job that "Socrates" did in the original. It is a constant within the example, though a variable when this is considered as an argument form. Here a might be called a free variable.
- (ii) We may consider separately that on which a quantifier operates, e.g. $Fx \supset Gx$. This is a "propositional function", it is one variety of incomplete expression. By itself it says nothing. The x in this might be called a free variable.
- (iii) Suppose that we have, at some step in an argument, ϕy , where y is an "arbitrarily selected individual", so that we shall be able to use U.G. and infer $(x) \phi x$. This y might be called a free variable.

These uses are all different, for x in (ii) and y in (iii) are variables within the example, whereas the a in (i) is not, while Fa in (i) and ϕy in (iii) are complete statements, whereas $Fx \supset Gx$ in (ii) is an incomplete statement. If the phrase "free variable" is to be used at all, it should be confined to just one of these three ways of using a symbol, perhaps most usefully to (ii).

III. Sets of symbols distinguished.

Having drawn these distinctions between ways in which a symbol may be used, we can remove various ambiguities by deciding to use a different set of symbols for each of these different functions. That is, we shall take one set of letters, say those from the beginning of the alphabet, a, b, c, d, e, f , and use one of these, but no others, as the symbol for any constant within the example. We shall take another set of letters, say those from the end of the alphabet, x, y, z, u, v, w , and use one of these, but no others, for any explicit variable within the example, that is for any bound variable or for the "free variable" in a propositional function. And we shall take a third set of letters, say

those from the middle of the alphabet, k, l, m, n, p, q, and use one of these, but no others, as the symbol for any "arbitrarily selected individual".

Thus if we have at any stage ϕa (or ϕb , etc.) we know that a represents what is within the example an actual named individual, so that we can apply E.G. to this formula but not U.G. On the other hand, if we have ϕk (or ϕl etc.) we know that k represents an "arbitrarily selected individual", and therefore that we can apply U.G. to this formula—though it would also be permissible to apply E.G. Any numbered step in an argument that contains a k (or l , etc.) represents a set of corresponding formulae in terms of a , b , c , etc., where these are all the individuals in the universe (or in the "universe of discourse" within which the argument occurs); if ϕx is any propositional function, then a numbered step ϕk indicates that ϕa , ϕb , ϕc , etc., could also be asserted at that point. We shall never have any propositional function, ϕx , as a numbered step in an argument.

IV. Re-statement of the rules.

We can now re-state the rules for the four procedures, U.I., U.G., E.G., and E.I., letting these distinctions between symbols take the place of restrictions.

U.I. can be applied to any universally quantified formula, and we can instantiate either to a genuine individual, such as a , or, by what I call U.I. (arb), to an arbitrarily selected individual such as k . What is true of all individuals can be asserted of any actual individual, or of a specimen individual that represents the whole range of actual ones. These two processes should be distinguished, although U.I. (arb) can be understood as an indication of the possibility of a set of applications of ordinary U.I. That is, we have as valid argument forms:

U.I. : $(x) \phi x; \therefore \phi a$

U.I. (arb): $(x) \phi x; \therefore \phi k$

Where ϕa is possible, so of course are ϕb , ϕc , etc., similarly where ϕk is possible, so are ϕl , ϕm , etc., and again wherever x occurs it could be replaced uniformly by y , z , etc. This will be assumed throughout, and in the statement of each valid argument form normally only one symbol from each set will be mentioned, but what is said will be taken to apply to any member of that set.

U.G. can be applied only to an "arbitrarily selected individual", *i.e.* we have the following valid argument form:

U.G.: ϕk ; $\therefore (x) \phi x$ (but *not* ϕa ; $\therefore (x) \phi x$)

Since we could go in two steps, by U.G. followed by U.I., from ϕk to ϕa , we could, but do not need to, add ϕk ; $\therefore \phi a$ as another variety of U.I.

Existential generalisation can be applied to any genuine individual, *i.e.* we have:

E.G.: ϕa , $\therefore (\exists x) \phi x$

Whereas in U.G. we must substitute x *uniformly* for k , we do not need to substitute x uniformly for a in E.G. For example, we can go from Raa ("a has a certain relation to itself") to $(\exists x) Rxa$ ("There is something which has that relation to a"). Also, since we can go from ϕk to ϕa and thence to $(\exists x) \phi x$, we can add ϕk ; $\therefore (\exists x) \phi x$ as another variety of E.G. Non-uniform substitution in this case is discussed below.

To state the rule for E.I., we take it that in working out any argument we have a list of symbols for constants $\{a, b, c, d, e, f, \}$ and that as each of these is used it is asterisked. By E.I. we can introduce any new constant, *i.e.* any symbol that has not so far been asterisked; thus we have:

E.I.: $(\exists x) \phi x$, $\{a^*, b^*, c^*, d, e, f, \}$; $\therefore \phi d$ (but *not* ϕa)

In my previous article I pointed out the ambiguity of such a formula as Rzw , when the letters are used as in Copi's system, without any distinctions between sets of symbols. This ambiguity could still be present in the formula Rka even with the distinctions just introduced. For if Rka were derived from $(\exists y) (x) Rxy$ by E.I. followed by U.I. (arb), it would represent such a set as Raa , Rba , Rca , *etc.*, *i.e.* it would represent a situation where one individual, a , had all individuals standing to it in the relation R . But if Rka were derived from $(x) (\exists y) Rxy$ by U.I. (arb.) followed by E.I., it would stand for such a set as Rab , Rbc , Rcd , *etc.*, where each individual stands in relation R to some individual, but perhaps to a different one in each case. To clear up this kind of ambiguity I suggest that the first sort of set only should be represented by Rka , and that the second sort of set should be represented by Rka_k . In the latter formula the subscript indicates that each of the individuals for which k stands has its own R -related individual. That is, a_k stands not for one individual but

for a set of individuals, whose individual names are symbolised by a_a, a_b, a_c , etc.

We shall, therefore, lay down the rule that ordinary E.I. cannot be applied to a formula that involves a symbol for an arbitrarily selected individual; instead, we can apply to such a formula what we shall call E.I. (arb), the rule for which may be stated as follows:

E.I. (arb): $(\exists x) \phi(k, x), \{a^*, b^*, c^*, d, e, f\}; \therefore \phi(k, d_x)$

In order to be able to use this to any purpose, we must allow corresponding moves in the reverse direction. We can, of course, go from $\phi(k, a_k)$ to $(x) \phi(x, a_x)$, by the rule already given for U.G. We cannot go from $\phi(k, a_k)$ to $(x) \phi(x, a)$, but it is obvious that this is not permissible, since the k would not have been uniformly replaced by x . But we can also simply reverse the E.I. (arb), and go from $\phi(k, a_k)$ to $(\exists x) \phi(k, x)$. Calling this E.G. (arb), we have:

E.G. (arb): $\phi(k, a_k); \therefore (\exists x) \phi(k, x)$

The meaning and justification of these processes, E.I. (arb) and E.G. (arb) may be illustrated by a symbolic proof of the validity of the argument "Everyone has a father, everyone's father is respected by him, therefore everyone has someone whom he respects". Taking "persons" as the universe of discourse, we can prove this as follows:

- | | |
|----------------------------------|------------------------------------|
| 1. $(x) (\exists y) Fyx$ | |
| 2. $(x) (y) \{Fxy \supset Ryx\}$ | / $\therefore (x) (\exists y) Rxy$ |
| 3. $(\exists y) Fyk$ | 1, U.I. (arb) |
| 4. $Fa_k k$ | 3, E.I. (arb) |
| 5. $Fa_k k \supset Rka_k$ | 2, U.I. |
| 6. Rka_k | 4, 5, M.P. |
| 7. $(\exists y) Rky$ | 6, E.G. (arb) |
| 8. $(x) (\exists y) Rxy$ | 7, U.G. |

We could give in words an exactly parallel demonstration of the validity of this argument as follows:

3. Take any Tom, Dick or Harry; by the first premise he has a father.

4. Call his father "Tom, Dick or Harry senior".

5, 6. By the second premise, Tom, Dick or Harry respects Tom, Dick or Harry senior.

7. So there is someone whom Tom, Dick or Harry respects.

8. Therefore everyone has someone whom he respects.

Thus the colloquial specimen individual, "Tom, Dick or

Harry" corresponds to our k , and the *correlated name* "Tom, Dick or Harry senior" corresponds to our a_k . In introducing a_k by E.I. (arb), we are as it were conferring the name "senior" on every father, but in such a way that William's father is to be "William senior", John's father is to be "John senior", and so on. By the interpretation already laid down for formulae containing k , step 4 represents the set of steps,

4. $Fa_a a$, 4. $Fa_b b$, 4. $Fa_c c$, etc.,

that is, the set "William senior is the father of William", "John senior is the father of John", and so on. Step 6 similarly represents the set of steps

6. Raa_a , 6. Rba_b , 6. Rca_c , etc.,

that is, the set "William respects William senior", and so on, and the move we have called E.G. (arb) represents the move from this set to the set represented by step 7,

7. $(\exists y)$ Ray, 7. $(\exists y)$ Rby, 7. $(\exists y)$ Rcy, etc.,

that is, the set "William has someone whom he respects" and so on. Since the move from each individual step in the set represented by 6 to the corresponding step in the set represented by 7 is valid (by ordinary E.G.), the move as a whole by E.G. (arb) is also valid. E.G. (arb) thus represents a set of ordinary E.G.'s, just as U.I. (arb) represents a set of ordinary U.I.'s.

Thus the symbol with a variable subscript, such as d_k , which is introduced by E.I. (arb) is neither a pure constant or individual name within the example nor a pure variable: it is a correlated name. It becomes a simple name for each value of the subscript; that is a_k is a correlated name, a_b is a simple name. Correlated names occur in ordinary speech, for example "Mrs. So-and-So" is like a_k while "Mrs. Smith" is like a_b ; and correlated definite descriptions are quite common, such as "each person's father", "the author of each book", and so on. Allowing for the difference between names and descriptions, "each person", "each book" correspond to our k , and "each person's father", "the author of each book" to our a_k .

A further complication is that if there is more than one symbol for an arbitrarily selected individual in an expression to which we apply E.I. (arb), the symbol to which we instantiate must have a subscript for each k , l , or whatever it may be. For example, if the universe of discourse were "points", and we

knew that for any two points there was a point half-way between them, we could call the point half-way between A and B "A-B-divider" and so on. In symbols, "A-B-divider" might become d_{ab} , and the argument could run:

1. $(x)(y)(\exists z) Hzxy$
2. $(y)(\exists z) Hzky$ 1, U.I. (arb)
3. $(\exists z) Hzkl$ 2, U.I. (arb)
4. $Hd_{kl}kl$ 3, E.I. (arb)

Step 4 here says "For points K, L, taken at random K-L-divider is half-way between K and L". The symbol d_{kl} is a doubly-correlated name, just as our coinage "K-L-divider" is. It may be compared with a doubly-correlated description, like "The Whato-Whichian War", considered as representing the whole set, "The Franco-Prussian War", "The Russo-Japanese War", and so on.

Moreover, since k itself could be regarded as a particular example of a_k , as a particular name correlated with k, we can argue by E.G. (arb) from, say, Rkk to $(\exists x) Rkx$ and hence to $(y)(\exists x) Ryx$. If, say, any arbitrarily selected individual resembles itself, then everything has something which it resembles. Thus E.G. (arb) allows for the fact that in generalising existentially we do not need to substitute *uniformly* for the symbol from which we generalise, even if this is the symbol for an arbitrarily selected individual.

V. Re-statement of conditional proof.

In my previous article I explained where the fallacy lies in an argument which, with the new convention about the use of letters, and with Γ as the sign for an assumption, we should write as follows:

- | | | |
|---|--------------------------------------|-----------|
| [| 1. ϕk | |
| | 2. $(x) \phi x$ | 1, U.G. |
| | 3. $\phi k \supset (x) \phi x$ | 1-2, C.P. |
| | 4. $(y) [\phi y \supset (x) \phi x]$ | 3, U.G. |
| | 5. $\phi a \supset (x) \phi x$ | 4, U.I. |

We can now put that explanation in other words, using the interpretation laid down at the start for formulae containing k. Step 1 means "Suppose that we can assert each of the set $\phi a, \phi b, \phi c$ etc". Step 2 says, correctly, that $(x) \phi x$ would follow from this supposition; but then step 3 infers that we can assert any

one of the set $\phi a \supset (x) \phi x$, $\phi b \supset (x) \phi x$, $\phi c \supset (x) \phi x$, etc., which does not follow at all. If we were validly to reach a step such as 3, which represents a set $\phi a \supset \dots$, $\phi b \supset \dots$, etc., we should have to start, not with $\Gamma 1. \phi k$ (which represents the assumption of a set of formulae) but with a set of assumptions $\Gamma 1. \phi a$, $\Gamma 1. \phi b$, etc., and of course step 2 of the invalid argument above would not follow from each of these.

Thus we see that the assumption in a conditional proof must not contain any symbol for an "arbitrarily selected individual". On the other hand, there are valid arguments in which we seem to need this, but where what we intend to indicate is a set of assumptions, $\Gamma 1. \phi a$, $\Gamma 1. \phi b$, etc. Take, for instance, the argument "All lawyers are clever, all respectable people are dull, therefore all respectable lawyers are both clever and dull". We could give a commonsense demonstration of its validity as follows:

"Suppose someone is a respectable lawyer; since he is a lawyer he must be clever, and since he is respectable he must be dull, so he is both clever and dull; this holds for any respectable lawyer, so all respectable lawyers are both clever and dull".

What we are doing here is to indicate that we could prove, by the appropriate supposition, about each individual in turn that if he is a respectable lawyer he is both clever and dull. What we are indicating could then be symbolised as follows, taking the "universe of discourse" as "persons":

1. $(x) [Lx \supset Cx]$
2. $(x) [Rx \supset Dx] \quad \therefore (x) [(Rx.Lx) \supset (Cx.Dx)]$
- | | | | |
|---|---|--|--------|
| $\left[\begin{array}{l} 3. \quad Ra. La \\ 10. \quad Ca. Da \end{array} \right.$ | $\left[\begin{array}{l} 3. \quad Rb. Lb \\ 10. \quad Cb. Db \end{array} \right.$ | $\left[\begin{array}{l} 3. \quad Rc. Lc \\ 10. \quad Cc. Dc. \end{array} \right.$ | } etc. |
|---|---|--|--------|
11. $(Ra. La) \supset (Ca. Da) \quad 11. (Rb. Lb) \supset (Cb. Db) \text{ etc.}$
12. $(x) [(Rx.Lx) \supset (Cx.Dx)] \quad 11, \text{ U.G.}$

The intermediate steps between 3 and 10, and their justifications, are quite straightforward and have been omitted. Each argument from an assumption at step 3 to a conclusion at step 10 is cashed, by the rule of conditional proof, for a hypothetical statement, and these are collected by U.G. to give the conclusion. This procedure is clearly valid, the only question is about how it should be symbolically presented.

Step 11, in terms of the standard meaning of k , can be written as 11. $(Rk. Lk) \supset (Ck. Dk.)$. But when we give just one version

of the steps from 3 to 10, it would be consistent to regard this whole sequence as a sub-argument within the main argument, and the individual we are arguing about as a constant within this sub-argument, although this sub-argument as a whole is simply a specimen from the point of view of the main argument. These steps should, therefore, be stated in terms of a symbol for an individual constant, such as a , and the argument should be written as follows, with the marginal note "Spec." to indicate that this assumption is itself a specimen one:

- | | | |
|-----|-----------------------------------|--|
| 1. | $(x) [Lx \supset Cx]$ | |
| 2. | $(x) [Rx \supset Dx]$ | $/ \therefore (x) [(Rx. Lx) \supset (Cx. Dx)]$ |
| 3. | $Ra. La$ | Spec. |
| 10. | $Ca. Da$ | |
| 11. | $(Rk. Lk) \supset (Ck. Dk)$ | 3-10, C.P. (arb) |
| 12. | $(x) [(Rx. Lx) \supset (Cx. Dx)]$ | 11, U.G. |

That is, we introduce, in addition to ordinary C.P., what we can call C.P. (arb), in which the assumption and the conclusion drawn from it are cashed for a hypothetical statement about an arbitrarily selected individual. This is permissible only if the assumption was itself a specimen one, and to be such it must be stated in terms of an individual constant that has not been previously used in this argument. This is necessary to ensure that in the deduction within the sub-argument nothing has been used that could not have been asserted about *any* individual.

This change of symbol at step 11 looks odd, but it can be easily explained by reference to what has been said earlier. At this point we step outside the sub-argument and consider it, as a whole, as a specimen. The change from a to k corresponds to the difference in function when "Socrates" is a constant within the example, but a variable when the example is considered from the outside as an example. Such a constant within the example is something which is implicitly quantified but only by a quantifier whose scope covers the argument as a whole. And a , in the sub-argument from step 3 to step 10, is precisely something which is implicitly quantified, but only by a quantifier whose scope covers this whole sub-argument. We are indicating that for any a , step 10 would follow from step 3. But the main argument looks at this sub-argument as a whole from the outside, taking it as a specimen, so from its point of view what the sub-argument is about is a variable. Thus step 11, which is

part of the main argument, is asserted about a variable; the implicit generality is now within this single numbered step, whereas there was no implicit generality *within* any single step of the sub-argument.

It is not necessary that the conclusion of the sub-argument, the consequent of the resulting hypothetical, should contain the symbol that changes from a to k : it could be any formula. Allowing for this, and using A , B , to represent any formulae, we can state the rules for ordinary C.P. and C.P. (arb) as follows:

C.P.: ($\Gamma A \rightarrow B$; $\therefore A \supset B$ (A does not contain k))

C.P. (arb): $\{a^*, b^*, c^*, d, e, f\}$, $\Gamma \phi d \text{ spec} \rightarrow \psi d$; $\therefore \phi k \supset \psi k$
 $\{a^*, b^*, c^*, d, e, f\}$, $\Gamma \phi d \text{ spec} \rightarrow B$; $\therefore \phi k \supset B$

VI. Summary and Conclusion.

The elementary valid argument forms concerning quantifiers may thus be summarised as follows:

U.I.: $(x) \phi x$; $\therefore \phi a$
 ϕk ; $\therefore \phi a$

U.I. (arb): $(x) \phi x$; $\therefore \phi k$

U.G. ϕk ; $\therefore (x) \phi x$

E.G.: ϕa ; $\therefore (\exists x) \phi x$
 ϕk ; $\therefore (\exists x) \phi x$

E.G. (arb): $\phi(k, a_k)$; $\therefore (\exists x) \phi(k, x)$

E.I.: $(\exists x) \phi x$, $\{a^*, b^*, c^*, d, e, f\}$; $\therefore \phi d$

E.I. (arb): $(\exists x) \phi(k, x)$, $\{a^*, b^*, c^*, d, e, f\}$; $\therefore \phi(k, d_k)$
 $(\exists x) \phi(k, l, x)$, $\{a^*, b^*, c^*, d, e, f\}$; $\therefore \phi(k, l, d_{kl})$

C.P.: $\Gamma A \rightarrow B$; $\therefore A \supset B$ (A does not contain k)

C.P. (arb): $\{a^*, b^*, c^*, d, e, f\}$, $\Gamma \phi d \text{ spec} \rightarrow \psi d$; $\therefore \phi k \supset \psi k$
 $\{a^*, b^*, c^*, d, e, f\}$, $\Gamma \phi d \text{ spec} \rightarrow B$; $\therefore \phi k \supset B$

This method of symbolisation gives us, at the cost of some multiplication of the names of the elementary operations, a system of natural deduction which satisfies almost completely the requirements for elegance set out in Section I. Every step is justified simply in terms of the step or steps from which it is derived, together with, in the case of E.I., E.I. (arb), and C.P. (arb), the list of used constants. Each line has an unambiguous meaning, and any valid deduction is valid in any line, granting of course that lines within the scope of an assumption may follow

only from that assumption, and not from the original premises alone. And the rules summarised above are obviously justified in terms of the meanings given to the three sets of symbols of which a , k , and x are the standard representatives.

This is, of course, not a new set of methods, but only a new way of writing the methods commonly used. The complications are due simply to the fact that we have explicitly recognised, in the symbols and in the names of the operations, the differences that are caused by the presence of an "arbitrarily selected individual". Argument in terms of such arbitrarily selected individuals is a central feature of natural deduction, and it is worth some complication to make it clear when we are using it and to understand why we are justified in using it in certain ways.

University of Otago

TOO GOOD A REASON TO BE A REASON

By S. A. GRAVE

MANY philosophers have thought it morally axiomatic that the best reason we could have for doing something is the conviction that we ought to do it. I shall try to show that their opinion is left undisturbed by Professor Nowell-Smith's contention that this is too good a reason.

"As in the case of 'because I thought it best', 'because I thought I ought to' does not give a reason for my choice; it gives too good a reason. But it indicates that the choice was a reasoned and not a casual one and, like 'because I thought it right', it indicates the type of reason, namely that it is a pro-attitude towards obeying a rule. . . ."¹

"Because I thought I ought to" does not give a reason for my decision since it leaves unspecified the rule to which I am subscribing. Similarly, "because I thought it best" does not say why I thought the course of action I chose better than any alternative, and this is what the questioner wants to know. Nowell-Smith's grounds for regarding "because I thought I ought to" as giving too good a reason for a decision are more

¹ *Ethics* (Pelican Books), p. 190.

conjectural, but if he is right in maintaining that the function of "I ought" is to express a certain kind of decision, he is right in maintaining that "because I thought I ought to" gives too good a reason for it. "'I ought,'" he says, "differs from 'I shall' in that, while 'I shall' can be used to express any decision, 'I ought' is only used to express decisions of a certain kind, namely those based on rules" (p. 261). If this is so, if "'I ought' entails 'I shall' of which it is a special case" (p. 268), the defect and excess which make it at once no reason for my decision and too good a reason are obvious: apart from its contextual implication, "I shall because I ought to" has the unanswerable and trivial finality of "I'm going to because I'm going to".

Is "I ought to, but I shall not" self-contradictory? It is, in Nowell-Smith's opinion, unless—knowing the infirmity of my will—I am coupling my decision with a prediction. The "contradictory character of 'I ought, but I shall not' is obscured by the fact that 'shall' also has another use in which it is not self-contradictory to say 'I ought, but I shall not' or logically odd to say 'I know I ought, but I wonder whether I shall'. This is the predictive use" (p. 268).

The term "logically odd" here needs some comment. According to Nowell-Smith's previous explanations, an utterance which is logically odd is not necessarily, not usually, logically absurd (pp. 83-85). With "ought" and "shall" both assigned decision-making functions, "I ought to, but I wonder whether I shall" meaninglessly expresses both decision and indecision. It is important to notice that logical oddity has become complete incoherence, since if one forgot that the "ought" is the "ought" of decision and remembered the kind of questions Nowell-Smith commonly describes as "logically odd" ("Why preferring peaches to apples, did you take a peach instead of an apple?") one might read into him an unintentional (though deceptive) agreement with the deontologists. Why is it logically odd to say "but shall I?" if not because, having said "I ought to", I have given myself a "logically impeccable" reason for saying "I shall"? (The agreement would be deceptive because of course the deontologists will not allow that a pro-attitude can supply a moral reason.)

Any doubt about the meaning of "logically odd" in this context is removed by the unambiguous language Nowell-Smith uses in a parallel case. "This is the right thing to do" expresses a decision and "indicates the type of reason" one has

for it. "This is the right thing to do, but shall I do it?" is "absurd", he says. And he uses its absurdity as an argument against the view that moral distinctions are factual distinctions: the sentence would not be absurd if "rightness" were a "property" (p. 152).

We are not always making a prediction when we say that we ought to do something or other but that we are not going to. We use these words or others to the same purpose when we know what we ought to do, don't want to do it and don't intend doing it. And if our words are to keep their plain meaning, "I ought" must be allowed another function besides that of expressing a decision as to how one shall act.

Nowell-Smith allows it another function, but one in which "I" takes on the externality of "you". He distinguishes between the "self-hortatory ought" of advocacy and the "judicial, verdict-giving ought", between the "ought" which we employ in making up our minds and the "ought" which expresses our minds made up. Hesitating between conflicting moral principles a man says to himself "I ought to do X", "I ought to do Y", or he opposes "I ought" to "I want". "In the first case it is quite natural to represent the two 'oughts' as being spoken by internal moral authorities advising or telling him what to do; and in the second to represent the conflict as one between the Voice of Conscience and Desire. But these are the voices of advocates, not of judges; and what they say is, not 'I ought', but 'you ought'." (pp. 261-262).

"You ought", when I am being spoken to by someone else, leaves me morally untouched until I say "I ought". When it is something I say to myself, it is either a mere idiosyncratic substitute for "I ought" (I am in the habit of addressing myself in the second person) or the difference of pronouns is significant, and "you ought" is a way of holding the acknowledgement of an obligation at arm's-length. When my conscience tells me what to do, I am telling myself what to do and not being told. I am acknowledging an obligation, therefore saying "I ought" or meaning it if the words I actually use happen to be "you ought".

There are philosophers, Nowell-Smith remarks, who will object to his distinction between the "self-hortatory 'you ought' and the verdict-giving 'I ought'" "on the grounds that, in the special case of conscience, the two are identical. My conscience, they will say, *is* myself." They hold, in Nowell-Smith's critical exposition of their views, that because of this identity, I act freely

only when I act in accordance with my conscience; when I do what my conscience condemns I am acted upon by my desires. The consequence is that no one does wrong voluntarily (pp. 263-264).

Nowell-Smith's distinction has a similar consequence. I may do something I believe to be wrong through lack of self-control, but I can never decide to do it, for I never say "I ought" without saying "I shall". It is a consequence he virtually acknowledges, in spite of his determination not to interfere with the data of moral theory. I can "think that I ought to be less greedy, vindictive or sanctimonious than I am . . . I cannot (logically) condemn any of these vices in myself while at the same time exercising them" (p. 311). Nowell-Smith and the critics he imagines as correcting him make the same kind of mistake. A man's conscience is no more an alien voice ordering him about than his desires are alien forces pushing him about. The voice of conscience is his own voice and what it says is not "you ought" but "I ought".

We might now look briefly at the "ought" of decision. Its difference from the "ought" of conscience is partly obscured, Nowell-Smith says, by the relative infrequency of moral struggles: "we have often only to recognize the 'you ought' of conscience to pass immediately to the 'I ought' of decision" (p. 262). What one actually finds in recalling the kind of situation Nowell-Smith has in mind is not "you ought" effortlessly becoming "I ought", but "I ought", since there is nothing to oppose it, eliding "I shall". No alternatives present themselves between which a decision has to be made.

Does "I ought" ever express a decision? It expresses a decision between conflicting moral principles, and the "ought" is then properly called "judicial" and "verdict-giving". Judges' decisions settle disputed questions and not, except incidentally, judges' indecisions. In making the verdict-giving "I ought" a special case of "I shall" Nowell-Smith conflates two senses of "decision" which he elsewhere¹ distinguishes: "To decide . . . is to determine a question, controversy or cause; its root meaning in English is the legal one. From this, by an easy transition, it comes to mean 'resolve, make up one's mind' . . ."

"I ought" would be too good a reason for a decision if it entailed "I shall". Since it is possible to choose to do what one believes to be wrong, "I ought" does not entail "I shall". It

¹ "Choosing, Deciding and Doing", *ANALYSIS*, Jan. 1958, p. 64.

may not give a reason for my decision, but its failure—if it fails—is not due to its expressing my decision. The deontologists may be mistaken, but they have not mistaken logical necessity for moral necessity.

University of Western Australia.

ON 'CONSCIENCE'

By NICHOLAS G. FOTION

ON page 171 of his *Ethics and the Moral Life* Bernard Mayo says:

So far I have suggested that the authority of conscience is ultimate and unchallengeable. For one can challenge an authority only by appeal to some other authority, so that one must rest somewhere with a decision to accept some ruling as final; and this decision will be a moral decision and conscience the final authority.

He goes on to cite examples in which one seemingly challenges the authority of conscience and shows, rightly, that in these examples the authority of conscience is not really being challenged at all. Sinning through moral weakness, for example, reflects a person's failure to live in accordance with a rule rather than a questioning of that rule as such:

The same is true if I try to comply with the rule but fail through incompetence or oversight, through failure to judge correctly what action is called for under the rule, in a particular situation. Even if I reject the rule itself, although it is true that I no longer regard it as authoritative, yet provided that I reject it in favour of some other rule, as is usually the case, the authority of conscience remains intact.

I should like to add two examples to Mayo's list. The first, like those above, only seems to be a case in which the authority of the conscience is challenged. It, along with Mayo's examples, is presented in order to form a contrast with the second example which, it will be argued, shows that the authority of one's conscience can be challenged (*i.e.* that there is a use of 'conscience' such that we can say 'I am challenging the authority of my own conscience').

Cases of the first type are common enough. Alpha feels that he is going against his conscience because he has told a white-lie to his very sick friend. Even though he may have

guilt feelings on telling this white-lie, and in this sense is going against his conscience, he is not challenging its authority since he decides to tell the white-lie in accordance with conscience. What Alpha has done actually is to replace the rule 'I ought not to tell a lie' with 'I ought to tell a lie' because of the special circumstances of the situation. The replacement of the one rule for the other was justified by an appeal to (by Alpha's making explicit) a higher order *moral* rule which directs a person not to cause another person great pain.

What Beta does is perhaps less common but not unheard of. He acts under the rule 'I ought to let another person make moral decisions for me'. At one time, in his youth, Beta fervently insisted that he ought to make his own moral decisions, but since that time, he has come to the realization that there are others who are much wiser than he. Before coming to this realization, he had been constantly criticized for many of his moral decisions. The irritating thing here was that only on occasion did Beta appreciate the points of their criticisms. Feeling himself inadequate, it was quite natural for Beta to seek out an adviser to help him out in these situations. At present his adviser actually does more than advise him; he guides and even leads Beta. On his part, Beta follows this leadership even though he differs with his adviser as to what he thinks the correct solution of certain moral questions might be. When Beta has these differences with his adviser, he usually expresses himself by saying 'It seems to me that this is the right thing to do but since my (learned and wise) adviser denies this is so, I must be wrong'.

Now the significant difference between what Alpha (and those who find themselves in the situations Mayo cites) and Beta are doing is that, in giving up the rule of his youth, there is doubt as to whether Beta is appealing to a (higher) *moral* rule at all; whereas it is clear that Alpha is appealing to such a principle. Beta reasons as follows: I ought to allow my friend to make my decisions for me, since he is wiser than I am; and one ought to let the wiser of two individuals make the decisions in such matters (because *whatever might be right or wrong* is more likely to be best established by the wiser of us). It is the above italicised words which indicate that Beta has come to no moral conclusions in reasoning the way he has, and consequently has not made an appeal to his conscience.

A further indication that Beta's decision is significantly different from Alpha's is that it does not lead directly to moral

solutions of moral questions. If a moral principle such as 'The general happiness ought to be maximized' or 'One ought not to cause undue pain to others' is adopted, one has a rule which at least allows him to deal with more particular moral questions. So Alpha tells his white-lie in order to avoid causing pain to another person. But from the fact that Beta has given up making moral decisions, nothing follows directly as a consequence. His adviser still has to make all the moral decisions and can adopt, therefore, whatever highest moral principles he sees fit.

It appears then that rather than speak of Beta's decision as a moral one, it is more accurate to speak of it as a judgment (decision) aimed at setting up the conditions for making a moral judgment (decision). This leaves it open for Beta to act contrary to the authority of his conscience on those occasions when his adviser tells him to do things which Beta would not have done had the decisions been his own. To say in such contexts something like 'It seems to me that this is the right thing to do, but since my adviser denies this is so, I must be wrong', is for Beta to concede that the self-directed commands he issues in accordance with moral rules (to paraphrase part of Mayo's use of 'conscience', p. 171) are not to be heeded and, strictly speaking, are no longer commands. For surely to challenge someone's authority is to say to him something like 'You can no longer command' or 'You can no longer decide on these matters' or, putting it somewhat misleadingly, 'Your decisions don't count'. In effect Beta is saying such things to himself, and in so far as he is, he is demonstrating that the authority of conscience is not unchallengeable.

It might be objected that since it is meaningful to say 'Beta did the right thing' in appealing to an adviser, Beta must be making a moral judgment and thus is acting in accordance with the authority of his conscience. One has only to ask 'But is this "thing" that Beta did a moral or procedural matter?' to see that this objection can be dealt with. A person can be said to act rightly when he adopts the correct method in coming to a decision or in making a moral judgment (*e.g.* one usually does the right thing procedurally in ethics when he waits until morning to make a weighty, but not pressing, decision), so a claim like 'Beta acted rightly' need not necessarily count against the position presented above. Nor need this position be upset by the appropriateness here of some comment like 'Beta is a good man' (in the sense that his actions reflect his attitude of trying to be good all the time) for one can be said to

be good simply because he tries to conform to rules set up by others. Such a person could want to be good, follow his adviser's lead, and yet feel that what his adviser tells him to do in some situations is wrong. And because he feels this way we can say that he is acting contrary to the authority of his conscience.

Coe College, Cedar Rapids, Iowa.

MALCOLM'S DREAMING

By K. STERN

PROFESSOR MALCOLM quotes Professor Austin as writing, 'There are recognized ways of distinguishing between dreaming and waking (how otherwise should we know how to use and contrast these words?)', and accuses him¹ of maintaining this, 'not because he (Austin) knows of any "recognized ways", but because he assumes he can know he is awake, and so must have some way of doing it'.

Putting aside the question, 'Can I know I am awake?' (Malcolm contends the sentence 'I know I am awake' is senseless), I do not see why Austin is committed by the quotation above to assuming that one can know whether one is awake. All Austin need mean here is that *when* we are awake, we may, if we like, contrast waking and dreaming experience, and that these two kinds of experience are so different that we may say either to ourselves or others, in the vast majority of cases, with perfect surety, 'That was a dream'.

But Professor Austin does, as a matter of fact, in lectures he gives in Oxford, produce some recognized ways of distinguishing between waking and dreaming. Most of the following ones are, I think, his. We talk of dreams having a 'dream-like quality'; a kind of muddled etherealness. For instance, rarely do we dream we smell or have tactile sensations. Hearing and seeing play the major roles in dreams, but even here, sounds and colours are not nearly so vivid or loud as in waking life. In waking life it is generally perceptions which are dominant. In dreams, feelings and emotions suffuse and dominate everything else. Dreams are very like story-telling. Sometimes as if we were

¹ *Dreaming* (Routledge and Kegan Paul, 1959), p. 114.

telling a story of someone else, sometimes of ourselves, and sometimes as if someone else were telling one about us. Like stories, dreams are relatively undetailed, any detail is relevant usually for effect. Dreams are sketched in with bold strokes. Things like weather, or specific undistinguished parts of time (afternoon), are not generally provided for. Perhaps the likeness of dreams to stories is the reason why in those rare instances when we do confuse dreams with reality, the confusion is usually between dreaming that something happened and thinking that the happening was related to us, and not so much between dreaming that something happened, and its having actually happened to us.

But how seldom this type of confusion occurs! Certainly its happening could never give us our ordinary concept of dreaming. So Malcolm is wrong when he writes (p. 66) 'When (someone) says, "I dreamt so and so", he implies, first, that it seemed to him on waking up as if the so and so had occurred . . .' Of course, when someone says, 'I dreamt so and so,' he never implies that it seemed to him on waking up as if the so and so had occurred, for if, on the contrary, it did seem to him as if the so and so had occurred, he should not have said, 'I dreamt that so and so occurred'. Malcolm insists (p. 51) that to find out that something was a dream is to find out that the incidents in question did not occur. But how often do we have to *find out* whether something was part of a dream or not? The question so rarely comes up just because dreams are so different from waking experience. Malcolm's denial that there are recognizable differences between waking and dreaming, which stems from his denial of the 'contents' of dreams, leads him to concentrate just on those very unusual times when there is a genuine confusion whether an incident was part of a dream. So Malcolm is led to write (p. 65) that he is inclined to believe that statements of the form 'I dreamt so and so' are always inferential in nature because one 'might always defend them as conclusions from certain facts or supposed facts, for, if you were asked how you know you dreamt so and so, you could always mention something you supposed proved or made probable that the thing in question did not occur and that therefore you dreamt it.' On the same page Malcolm tells us that the reason anything is possible in a dream is, 'If we know that it is impossible for a certain thing to have occurred, then the waking impression that it occurred is false, and we know therefore that we dreamt the impossible thing.' As if, on awakening, I need first remember

that what I dreamt was impossible in order to know I must have dreamt it! 'I dreamt so and so' is seldom ever a conclusion, and almost never one from the non-existence of a supposed fact. The facts which lead us to use the locution 'I dreamt . . .' are the facts which distinguish waking from dreaming.

Sometimes Malcolm talks of the expression 'I have the impression (upon arising) that p occurred'. But this expression has, in ordinary language, the sense of an attenuated 'I think that p occurred'. But Malcolm thinks that, in general, when we awake we have the impression that p occurred. This is, of course, false. On page 65 Malcolm attempts to change the ordinary sense of 'having the impression that p occurred', and make it uncommitted as to the occurrence of p. 'What can have no justification and requires none is your statement that you have the *impression* that so and so occurred. (You may or may not believe that it did occur)' (*italics Malcolm's*). Suppose there is such a use of the expression 'having the impression that p occurred' (which I doubt). Then, if people ever are in such a state when awakening, why should they say, 'I am under the impression that p occurred', if they did not have reasons for thinking that perhaps so and so occurred in a dream? And such reasons would have to be other than the discovery that so and so did not occur, for if they *had* that reason, then they could not, in Malcolm's sense of the phrase, be under the impression that p occurred. Anyway, what could Malcolm have in mind when he says that you may have the impression that so and so occurred and at the same time you may or may not believe it occurred? Either: (1) 'I have the impression that p occurred' has the sense 'I dreamt that p occurred', which implies 'p did not occur'; or (2) 'I have the impression that p occurred' has the usual sense, 'I think p occurred'; or (3) 'I have the impression that p occurred' has the sense 'I am not sure whether or not p did occur'. But this last (3) is not *independent* of my belief about the occurrence of p. I may believe p occurred. Suppose I do. Then, 'I have the impression p occurred' cannot be anything like 'I dreamt that p occurred'. I may not believe p occurred. Suppose I do not. Then, 'I have the impression that p occurred' will imply 'I dreamt that p occurred'. I may not be sure whether or not p occurred. Suppose I am not. Then, 'I have the impression that p occurred' has something like the sense 'I don't know whether I dreamt p or not'.

Malcolm thinks that this third case is the paradigm one of arising after a dream. But it is not. It is the first which is. And

this is the case in which we can tell just from the nature of the dream experience, i.e. by recognized ways. It is just implausible to say that finding out that 'I dreamt that p occurred' comes to finding out that p did not occur.

University of Birmingham.

STERN'S DREAMING

By NORMAN MALCOLM

1. It is pretty probable that Professor Austin meant that one can decide whether one *is* (not just *was*) dreaming or awake. The remainder of the sentence quoted by me and Mr. Stern is: 'and of deciding whether a thing is stuffed or live, and so forth'. Austin is saying that there are 'recognized procedures' for allaying various sorts of doubts, including a doubt whether one is awake or asleep and whether a certain thing is stuffed or live; and he is implying that one can find out both that one is awake and that the thing is stuffed. But let Austin speak for himself.

2. Stern misreads me and so barks up the wrong tree. I do not deny that there are general differences between dreams and waking experiences. It may be true that generally there isn't any weather in dreams: I don't recall any in mine. What I deny is that one can make use of Stern's 'recognizable differences' or of any 'dream-like quality' to determine that one is now awake or now dreaming. Also I agree that commonly one does not have to *find out* whether something was a dream. One usually knows it without having actually inferred it. It appears to me, however, that one should always be able to give grounds, upon demand, for saying it was a dream and not reality. That is all I meant by 'inferential', and I may be wrong but Stern offers no evidence of it. 'Because it was so muddled and ethereal' offers grounds and appeals to the fact that normal waking life isn't like that.

3. I regret that I cannot follow Stern's remarks about impressions. If he thinks that 'It seemed to me as if so-and-so was happening' always implies that the speaker believed that the so-and-so was happening, then he is mistaken. Once when I saw a display of Northern Lights it seemed to me as if the heavens were on fire although I knew they weren't. The expression 'I had the impression that X occurred' is often used, like the expressed 'It seemed to me as if X occurred', in a sense in which it is not implied that the speaker believed that X occurred.

Cornell University

A NOTE ON SØRENSEN AND EXISTENCE

By ROBERT BINKLEY

MR. SØRENSEN¹ charges Russell and Quine with a mistake in their accounts of existence; they both retain "*exist* as a predicate of non-linguistic entities" (p. 129). But *if* this is a mistake in Russell (and Quine) then Sørensen also commits it.

I paraphrase the argument against Russell: Russell says that since "The golden mountain does not exist" simply means "There is no entity *c* such that '*x* is golden and mountainous' is true when *x* is *c*, but not otherwise", the contradiction involved in saying "The golden mountain does not exist" is resolved. But, "There is no entity *c* such that . . ." means "An entity *c* such that . . . does not exist", and this is as contradictory as what we began with since it implies that an entity *c* such that . . . does exist—as a subject of discourse.

Sørensen's own view is that *exist* is a predicate of signs. "*A is (not)*" is the equivalent of "*A* denotes (does not denote)" (p. 127). Further, he insists that "to denote" expresses a relation. Consequently, what he offers in place of "*A is*" is incomplete in the way in which "*A is to the right of*" is incomplete. An appropriate completion would be "*A* denotes an entity". Relations have converses, and so this is equivalent to "An entity is denoted by '*A*'". Similarly, the denial of existence would yield "No entity is denoted by '*A*'".

Now Sørensen's anti-Russell argument can be turned against him, for he says that since "*A does not exist*" simply means "No entity is denoted by '*A*'", the contradiction involved in saying "*A does not exist*" is resolved. But, "No entity is denoted by '*A*'" is equivalent to "There is no entity *c* such that *c* is denoted by '*A*'". (If Sørensen resists this step, he should allow Russell to say simply "No entity *c* is such that . . ." which would avoid even the appearance of a difficulty.) And this just means "An entity *c* such that *c* is denoted by '*A*' does not exist". And this is as contradictory as what we began with since it implies that an entity denoted by '*A*' does exist—as a subject of discourse.

Of course, really, neither of these arguments is any good. There is no reason to suppose that "An *A* such that it is *B* does not exist" implies that an *A* such that it is *B* does exist—even as a subject of discourse. No reason except, perhaps, the ancient superstition that for every significant sentence we must be able to mention an object that the sentence is about.

Duke University

¹ "An Analysis of 'To Be' and 'To Be True'," ANALYSIS 19.6 (June 1959), pp. 122-31.

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